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Analyzing the Adomian Modified Decomposition Method for the Study of Free Vibrations in Rotating Beams

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Abstract

This work uses the Adomian modified decomposition method (AMDM) to analyze the dynamic behavior of a rotating Euler-Bernoulli beam subjected to different boundary conditions. The principal differential equation controlling the beam's rotation is transformed into a recursive algebraic equation by using AMDM. The appropriate mode shapes and dimensionless natural frequencies may be readily derived concurrently using the boundary condition equations. Here we show the calculated outcomes for a variety of boundary conditions, offset lengths, and rotating speeds. The reported results of the convergence and comparison tests guarantee the correctness. An efficient and precise technique for free vibration analysis of spinning beams with arbitrary boundary conditions is shown by the AMDM.

Introduction

The rotating Euler-Bernoulli beams have been the subject of numerous investigations because they are widely used in various aeronautical, robotic, and helicopter blade and wind turbine engineering fields. The free vibration analysis of rotating beams has been extensively studied by many researchers [1–10] with great success. Different numerical or analysis methods such as differential transformation method [1, 2], the Frobenius method [3], finite element method [4, 5], and dynamic stiffness method [6] have been used in solving free vibration problems of such structures. References in [4, 5] give an exhaustive literature survey on the free vibration analysis of rotating beams. References in [11–13] discussed dynamic response of rotating beams with piezoceramic actuation and localized damages. No attempt will be made here to present a bibliographical account of previous work in this area. Few selective recent papers [1–10] which provide further references on the subject are quoted.

Until now, most of the vibration analysis of rotating beams has been limited to classical boundary conditions (i.e., which are either clamped, free, simply supported, or sliding). In practice, however, the characteristics of a test structure may be very well depart from these classical boundary conditions. In this paper, a relatively new computed approach called Adomian modified decomposition method (AMDM) [14–21] is used to analyze the free vibration for the rotating Euler-Bernoulli beams under various boundary conditions, rotating speeds, and offset lengths. The AMDM is a useful and powerful method for solving linear and nonlinear differential equations. The goal of the AMDM is to find the solution of linear and nonlinear, ordinary, or partial differential equation without dependence on any small parameter like perturbation method. The main advantages of AMDM are computational simplicity and do not involve any linearization, discretization, perturbation, or unjustified assumptions which may alter the physics of the problems [14]. In AMDM, the solution is considered as a sum of an infinite series and rapid convergence to an accurate solution [15]. Recently, AMDM has been applied to the problem of vibration of structural and mechanical systems, and this method has shown reliable results in providing analytical approximation that converges rapidly [16–21].

Using the AMDM, the governing differential equation for the rotating beam becomes a recursive algebraic equation [14–17]. The boundary conditions become simple algebraic frequency equations which are suitable for symbolic computation. Moreover, after some simple algebraic operations on these frequency equations, we can obtain the natural frequency and corresponding closed-form series solution ofmode shape simultaneously. Finally, some numerical examples are studied to demonstrate the accuracy and efficiency of the proposed method.

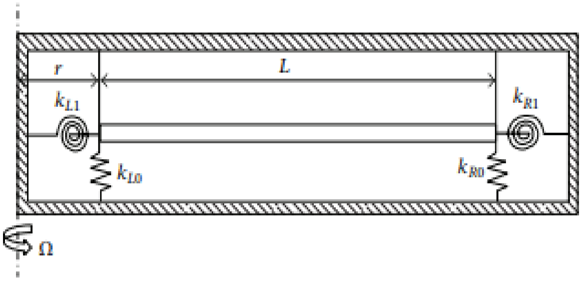


Figure 1: A rotating beam elastically restrained at both ends.

. AMDM for the Rotating Beams Consider the free vibration of a rotating Euler-Bernoulli beam with length L, constant thickness h, and width b, as shown in Figure 1.The partial differential equation describing the free vibration of a rotating beam is as follows [1, 2]:

$$EI\frac{d^4 w(x,t)}{dx^4} + \rho A\frac{d^2 w(x,t)}{dt^2} - \frac{d}{dx}\left[T(x)\frac{dw(x,t)}{dx}\right] =$$

where$E$ is Young's modulus, $I(x) = bh3\ /12$ is the crosssectional moment of inertia of the beam, $A = bh$ is the crosssectional area, and ρ is the density of the beam. (x) is the axial force due to the centrifugal stiffening and is given by the following:

$$T(x) = \int_x^L \left[\rho A\Omega^2 (r+x)\right] dx$$
$$= 0.5\rho A\Omega^2 \left(L^2 + 2rL - 2rx - x^2\right),$$

where Ω is the angular rotating speed of the beam and r is offset length between beam and rotating hub. According to modal analysis approach (for harmonic free vibration), the $(x,)$ can be separable in space and time:

$$w(x,t) = \phi(x)\, e^{i\omega t},$$

where$i = \sqrt{-1}$, (x) and ω are the structural mode shape and the natural frequency, respectively. Substituting (3) into (1), then separating variable for time t and space x, the ordinary differential equation for the rotating beam can be obtained:

$$EI\frac{d^4\phi(x)}{dx^4} - \frac{d}{dx}\left[T(x)\frac{d\phi(x)}{dx}\right] - \rho A\omega^2\phi(x) = 0.$$

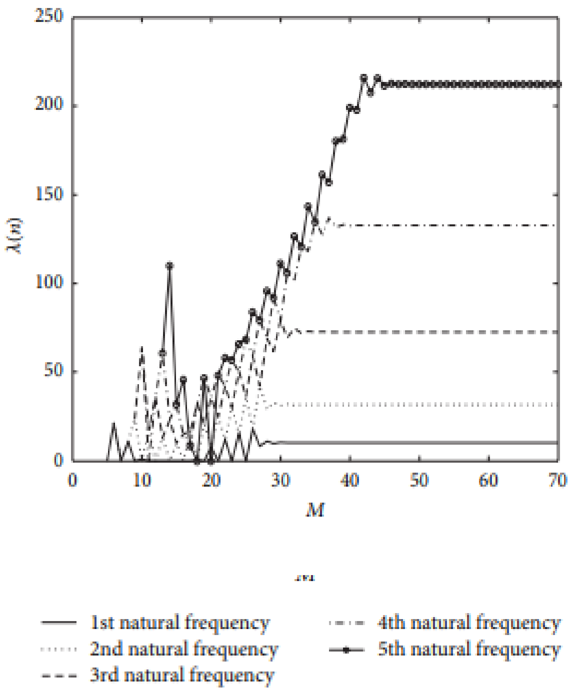


Figure 2:The first five dimensionless natural frequencies $\lambda(n)$ as the function of the series summation limit M.

Substituting (2) into (4), then rewriting (4) in dimensionless form

$$\frac{d^4\Phi(X)}{dX^4} - 0.5U^2(1+2R)\frac{d^2\Phi(X)}{dX^2}$$
$$+ U^2 R\frac{d}{dX}\left[X\frac{d\Phi(X)}{dX}\right] + 0.5U^2\frac{d}{dX}\left[X^2\frac{d\Phi(X)}{dX}\right]$$
$$- \lambda^2\Phi(X) = 0,$$

where$X = x/L$, $\Phi(X) = \phi(x)/L$, $R = r/L$, $U = \sqrt{\rho A\Omega 2L4/EI}$ is the dimensionless rotating speed and $\lambda = \sqrt{\rho A0\omega 2L4/EI}$ is the dimensionless natural frequency. According to the AMDM [11–18], $\Phi(X)$ in (5) can be expressed as an infinite series:

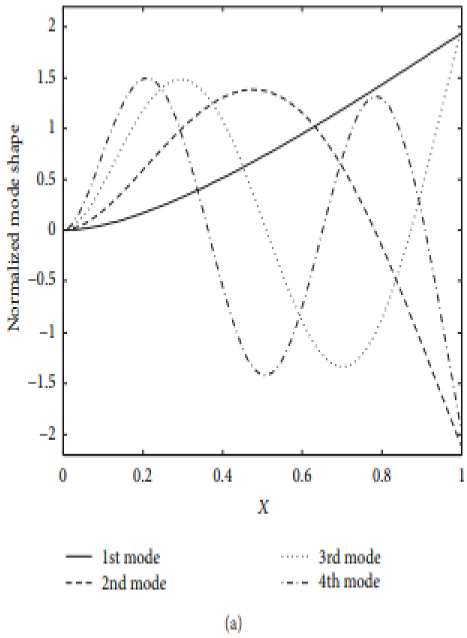
$$\Phi(X) = \sum_{m=0}^{\infty} C_m X^m,$$

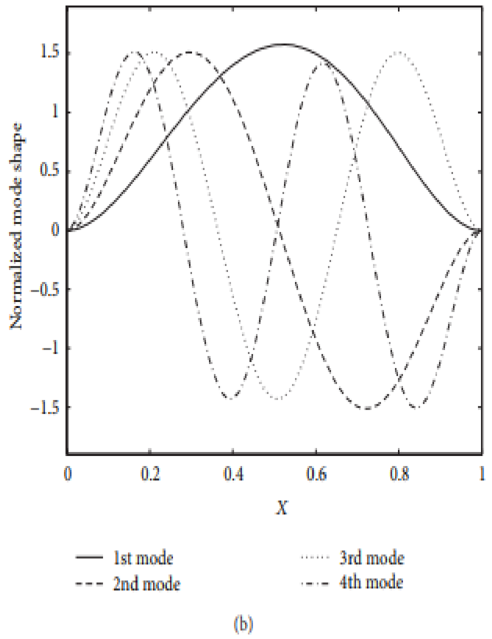
where the unknown coefficientsCm will be determined recurrently.

Impose a linear operator $G=d4\ /dX4$, then the inverse operator of G is therefore a 4-fold integral operator defined by the following

$$G^{-1} = \int\int\int\int_0^x (\cdots)\,dX\,dX\,dX\,dX,$$

$$G^{-1}G[\Phi(X)] = \Phi(X) - C_0 - C_1X - C_2X^2 - C_3X^3.$$





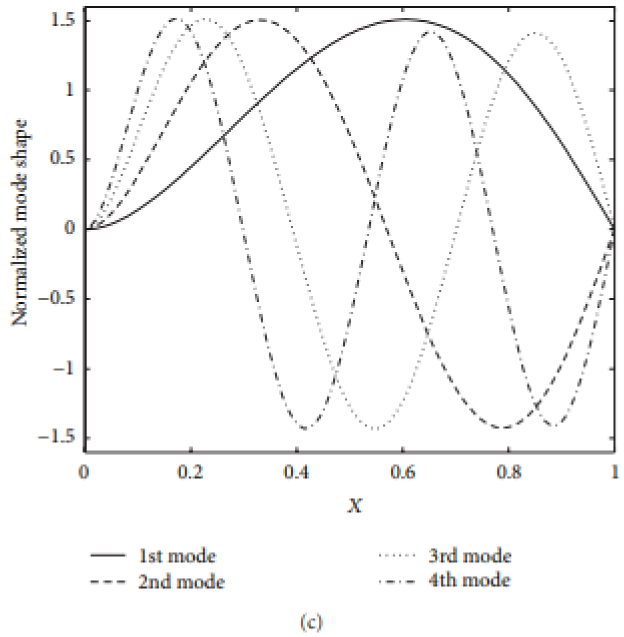


Figure 3: The first four normalized mode shapes for the (a) clamped-free beam, (b) clamped-clamped beam, and (c) clamped-simply supported beam when dimensionless rotating speed U=4 and offset length R=3.

Applying both sides of (5) with G−1, we get the following:

$$G^{-1}G[\Phi(X)] = -G^{-1}\left\{-0.5U^2(1+2R)\frac{d^2\Phi(X)}{dX^2} + U^2R\frac{d}{dX}\left[X\frac{d\Phi(X)}{dX}\right] + 0.5U \times \frac{d}{dX}\left[X^2\frac{d\Phi(X)}{dX}\right] - \lambda^2\Phi(X)\right.$$

Substituting (6) and (8) into (9), we get the following:

$$\Phi(X) = \sum_{m=0}^{3} C_m X^m + \sum_{m=0}^{\infty} \frac{0.5U^2(1+2R)(m+1)(m+2)C_{m+2}}{(m+1)(m+2)(m+3)(m+4)} X^{m+} - \sum_{m=0}^{\infty} \frac{(m+1)^2U^2RC_{m+1}}{(m+1)(m+2)(m+3)(m+4)} X^{m+4}$$

$$- \sum_{m=0}^{\infty} \frac{0.5U^2m(m+1)C_m}{(m+1)(m+2)(m+3)(m+4)} X^{m+4} + \frac{\lambda^2 C_m}{(m+1)(m+2)(m+3)(m+4)} X^{m+4}.$$

Finally, the coefficients Cm in (10) can be determined by using the following recurrence relations:

$$C_0 = \Phi(0), \qquad C_1 = \frac{d\Phi(0)}{dX}$$

$$C_2 = \frac{1}{2}\frac{d^2\Phi(0)}{dX^2}, \qquad C_3 = \frac{1}{6}\frac{d^3\Phi(0)}{dX^3} \quad (1$$

$$C_{m+4} = \frac{0.5U^2(1+2R)C_{m+2}}{(m+3)(m+4)} - \frac{(m+1)U^2RC_{m+1}}{(m+2)(m+3)(m+4)} - \frac{0.5U^2mC_m}{(m+2)(m+3)(m+4)} + \frac{\lambda^2 C_m}{(m+1)(m+2)(m+3)(m+4)}, \quad m \geq 0. \quad (1$$

We may approximate the above solution by the M-term truncated series, and (6) can be rewritten as follows:

$$\Phi(X) = \sum_{m=0}^{M} C_m X^m.$$

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Equation (13) implies that $\sum\infty\ m=M+1\ CmXm$ is negligibly small. The number of the series summation limit M is determined by convergence requirement in practice.

From the above analysis, it can be found that there are five unknown parameters ($C0$, $C1$, $C2$, $C3$, and λ) for the free vibration analysis of the rotating beam. These unknown parameters can be determined by using the boundary condition equations of the beam, and then the natural frequencies and corresponding mode shapes for the rotating beams can be obtained.

Results and Discussion

In order to verify the proposed method to analyze the free vibration of the rotating beam shown in Figure 1, several numerical examples will be discussed in this section.

As mentioned earlier, the closed-form series solutions of mode shape functions in (13) will have to be truncated in numerical calculations. It is important to check how rapidly the dimensionless natural frequencies (n) computed

TABLE 2: The first five dimensionless natural frequencies $\lambda(n)$ for a clamped-clamped beam with different dimensionle and offset lengths R.

R	U	Methods	Mode index n: 1	2	3	4	
0	0	Present	**22.373285**	**61.672823**	**120.903392**	**1.998594481274627**	2
		[3]	22.3733	61.6728	120.9034	–	
	1	Present	**22.465244**	**61.801647**	**121.044116**	**200.006494**	
		[3]	22.4652	61.8016	121.0441	–	
	2	Present	**22.738323**	**62.186191**	**121.465166**	**200.446898**	
		[3]	22.7383	62.1862	121.4652	–	
	4	Present	**23.791502**	**63.696418**	**123.132818**	**202.197641**	
		[3]	23.7915	63.6964	123.1328	–	
1	1	Present	**22.601469**	**61.987491**	**121.248139**	**200.220805**	
		[3]	22.6015	61.9875	121.2481	–	
	2	Present	**23.269013**	**62.919871**	**122.275462**	**201.300307**	
		[3]	23.2690	62.9199	122.2755	–	
	4	Present	**25.721997**	**66.488904**	**126.285440**	**205.551660**	
		[3]	25.7220	66.4889	126.2854	–	
2	1	Present	**22.736642**	**62.172635**	**121.451744**	**200.434838**	
		[3]	22.7366	62.1726	121.4517	–	
	2	Present	**23.784414**	**63.642885**	**123.079231**	**202.149350**	
		[3]	23.7844	63.6429	123.0792	–	
	4	Present	**27.477304**	**69.138595**	**129.343258**	**208.839952**	
		[3]	27.4773	69.1386	129.3433	–	
3	1	Present	**22.870783**	**62.3570858594266**	**121.6549319912815**	**200.6485958268648**	2
		[3]	22.8708	62.3571	121.6549	–	
	2	Present	**24.285626**	**64.355627**	**123.876600**	**202.994081**	
		[3]	24.2856	64.3556	123.8766	–	
	4	Present	**29.093946**	**71.663126**	**132.313123**	**212.065591**	
		[3]	29.0939	71.6631	132.3131	–	

through AMDM converge toward the exact value as the series summation limit M is increased. To examine the convergence of the solution, a clamped-free beam with dimensionless rotating speed U=4 and dimensionless offset length R=3 is considered. In this study, the classical boundary conditions (such as clamped, simply supported, and free) can be considered as the special cases of (14) and (15). For example, the clamped boundary condition is obtained by setting the stiffness of the translational and rotational springs to be extremely large (which is represented by a very large number, 1×109 , in this paper). Similarly, for simply supported boundary condition, the stiffness of the translational and rotational springs is set to 1 × 109 and 0, respectively. For free boundary condition, the stiffness of the translational and rotational springs is set to 0. Figure 2 shows the first five dimensionless natural frequencies (n) as the function of the series summation limit M. Clearly, the (n) converges very quickly as the series summation limit M is increased. The excellent numerical stability of the solution can also be found in Figure 2.

For brief, the series summation limit M in (13) will be simply truncated to M = 60 in all the subsequent calculations. The dimensionless natural frequencies (n) are kept accurate to the sixth decimal place for comparison purpose. Tables 1, 2, and 3 list the first five dimensionless natural frequencies (n) of the beam under various dimensionless rotating speeds U and offset lengths R for clamped-free, clamped-clamped, and clamped-simply supported boundary conditions, respectively. Those calculated results are compared with those listed in [1, 3, 4], and excellent agreement is found. Figure 3 shows the first four normalized mode shapes for different boundary conditions when dimensionless rotating speed U=4 and offset length R=3.

Figures 4 and 5 show the first five dimensionless natural frequency ratios $(n)/\lambda 0(n)$ for the clamped-free beam as the functions of the dimensionless rotating speed U and offset length R, where $\lambda 0(n)$ is the corresponding dimensionless natural frequencies when U=0 (nonrotating beam). From Figures 4 and 5, it can be found that the natural frequencies' ratios increase when the rotating speed or offset length increases for both beams. However, the variations on the natural frequency ratios of the low order modes are more sensitive to the rotating speed or offset length.

Next, the beams with general boundary conditions are discussed. Because the proposed method based on AMDM technique offers a unified and systematic procedure for

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Table 3: The first five dimensionless natural frequencies $\lambda(n)$ for a clamped-simply supported beam with different di speeds U and offset lengths R.

R	U	Methods	Mode index n 1	2	3	4
0	0	Present	**15.418206**	**49.964862**	**104.247696**	**178.269729**
		[3]	15.4182	49.9649	104.2477	—
	1	Present	**15.512970**	**50.093465**	**104.388569**	**178.416902**
		[3]	15.5130	50.0935	104.3886	—
	2	Present	**15.793333**	**50.476967**	**104.809884**	**178.857595**
		[3]	15.7933	50.4770	104.8099	—
	4	Present	**16.861201**	**51.977798**	**106.475988**	**180.608162**
		[3]	16.8612	51.9778	106.4760	—
1	1	Present	**15.650431**	**50.276757**	**104.591434**	**178.630496**
		[3]	15.6504	50.2768	104.5914	—
	2	Present	**16.324050**	**51.198754**	**105.614686**	**179.707690**
		[3]	16.3240	51.1988	105.6147	—
	4	Present	**18.739775**	**54.700871**	**109.594297**	**183.942227**
		[3]	18.7398	54.7009	109.5943	—
2	1	Present	**15.786476**	**50.459230**	**104.793819**	**178.843781**
		[3]	15.7865	50.4592	104.7938	—
	2	Present	**16.834938**	**51.908185**	**106.412044**	**180.552930**
		[3]	16.8349	51.9082	106.4120	—
	4	Present	**20.412962**	**57.264050**	**112.606262**	**187.203776**
		[3]	20.4130	57.2641	112.6063	—
3	1	Present	**15.921144**	**50.640895**	**104.995728**	**179.056759**
		[3]	15.9211	50.6409	104.9957	—
	2	Present	**17.327885**	**52.605804**	**107.202130**	**181.393382**
		[3]	17.3279	52.6058	107.2021	—
	4	Present	**21.932288**	**59.690141**	**115.520706**	**190.396605**
		[3]	21.9323	59.6901	115.5207	—

vibration analysis, the modification of boundary conditions from one case to another is as simple as changing the values of the stiffness of translational and rotational springs. And it does not involve any changes to the solution procedures or algorithms.

Table 4 lists the first five dimensionless natural frequency (n) for the beam with different dimensionless rotating speeds U and different rotational springs $KL1$ and $KR1$ when the translational springs $KL0 = KR0 = 1 \times 109$ and the dimensionless offset length R=3. From Table 4, it is found that the natural frequencies increase when the offset length or rotating speed increases, as expected. Figure 6 shows the first four normalized mode shapes of the rotating beam listed in Table 4. From Figure 6, it can be found that the discrepancies of the mode shapes under different rotating speeds are very small. However, the natural frequencies are quite different, as shown in Table 4.

Based on the developments achieved and results obtained in this paper, the following remarks can be made.

(1) The essential steps of the AMDM application includes transforming the governing differential equation for the rotating beam into algebraic equation; by using the boundary condition equations, any desired dimensionless natural frequencies and corresponding mode shapes can be easily obtained simultaneously.

(2) All the steps of the AMDM are very straightforward, and the application of the AMDM to both equations of motion and the boundary conditions seems to be very involved computationally. However, all the algebraic calculations are finished quickly using symbolic computational software (such as MATLAB). Besides all these, the analysis of the convergence of the results shows that AMDM solutions converge fast. The results of the AMDM are found in excellent agreement with available published results.

Conclusions

In this paper, free vibrations of the uniform rotating Euler-Bernoulli beams under different boundary conditions are analyzed using Adomian modified decomposition method (AMDM). The advantages of the AMDM are its fast convergence of the solution and its high degree of accuracy. Natural frequencies and corresponding mode shapes with various boundary conditions, dimensionless offset length, and dimensionless rotating speed are presented.

Table 4: The first five dimensionless natural frequencies $\lambda(n)$ for the beam with different dimensionless offset lengths R and rotational spring stiffness K_{L1} and K_{R1} when the translational spring stiffness $K_{L0} = K_{R0} = 1 \times 10^9$ and the dimensionless offset length $R = 3$.

U	K_{L1}	K_{R1}	Mode index n 1	2	3	4	5
1	0	0	10.728125	40.379864	89.736290	158.826493	247.654314
	10	20	18.722928	52.557379	104.894264	176.148714	266.614189
	100	200	22.240871	60.660903	118.406571	195.403654	291.714050
2	0	0	12.901808	42.950832	92.403230	161.529064	250.373747
	10	20	20.251238	54.708735	107.278904	178.653087	269.187647
	100	200	23.660610	62.669418	120.641412	197.764434	294.153770
3	0	0	15.741169	46.861509	96.655335	165.918965	254.831087
	10	20	22.506559	58.074886	111.115686	182.735812	273.412896
	100	200	25.806448	65.847359	124.257999	201.625455	298.167186
4	0	0	18.856444	51.730723	102.251676	171.841633	260.921080
	10	20	25.233188	62.406770	116.227213	188.272249	279.199963
	100	200	28.463584	69.993005	129.113530	206.884873	303.679326

Furthermore, the natural frequencies obtained by using AMDM are in excellent agreement with published results.

It should be noted that the proposed method can be used to analyze the vibration of the rotating beams under arbitrary boundary conditions. The vibration analysis for different boundary conditions and/or rotating speed is as simple as changing the value of corresponding parameters and does not involve any changes to the solution procedures or algorithms.

The results in this paper show that the AMDM technique is reliable, powerful, and promising for solving free vibration problems for rotating beams. The author believes that the AMDM can further be applied to the Timoshenko rotating beam problems and also it can be used as an alternative to other solution techniques such as finite element method, differential quadrature method, and Frobenius method.

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